Benha University
Faculty Of Engineering at Shoubra

ECE 122
Electrical Circuits (2)(2016/2017)
Lecture (10)
Transient Analysis (P1)

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Reference Chapter 16

Schaum's Outline Of Theory And Problems Of Electric Circuits

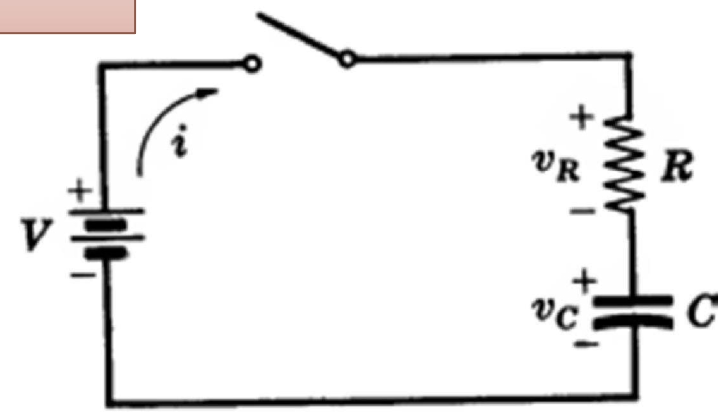
https://archive.org/details/TheoryAndProblemsOfElectricCircuits

1^{st} Order R-C

DC

First-Order RC Transient Step-Response

- Assume the switch S is closed at t = 0
- Apply KVL to the series RC circuit shown:



$$\frac{1}{C}\int i\,dt + Ri = V$$

- Differentiating both sides which gives:

$$\frac{i}{C} + R\frac{di}{dt} = 0 \quad \text{or} \quad \left(D + \frac{1}{RC}\right)i = 0$$

- The solution to this homogeneous equation consists of only the complementary function since the particular solution is zero.
- To find the complementary Solution, solve the auxiliary equation:

$$m + \frac{1}{RC} = 0 \qquad m = \frac{-1}{RC} = \frac{-1}{\tau} \qquad \tau = RC$$

Time constant

The complementary Solution is :

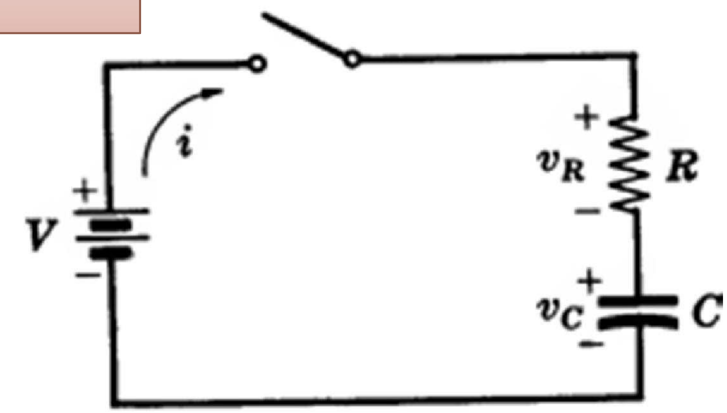
$$i = Ae^{mt} \qquad i = Ae^{\frac{-t}{\tau}}$$

First-Order RC Transient Step-Response

- To determine the constant "A" we note that :

at $t = 0$ is $Ri_0 = V$ or $i_0 = V/R$.

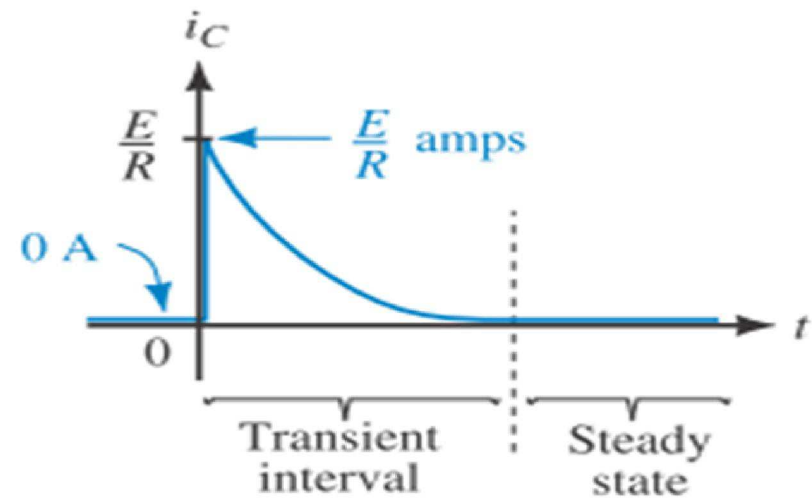
Where Vc (0) = 0



- Now substituting the value of i_o into current equation
- We obtain A = V/R at t = 0.

$$i = \frac{V}{R} e^{-t/RC}$$

has the form of an exponential decay starting from the transient value to the final steady-state value of 0 ampere in 5 time-constants



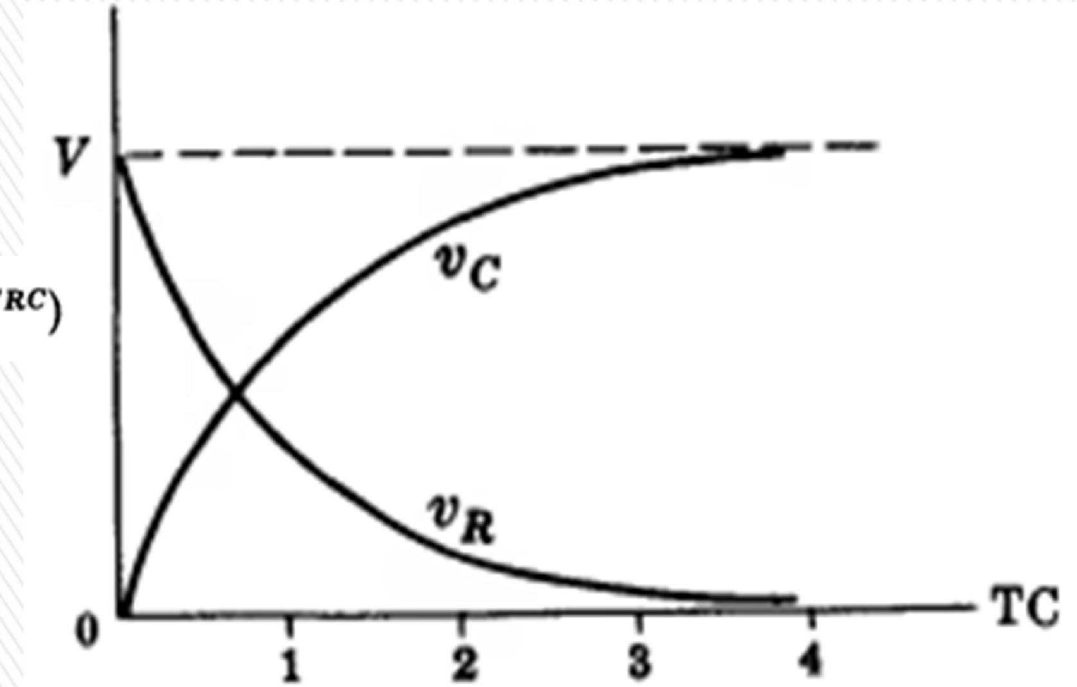
First-Order RC Transient Step-Response

- The voltage across the resistor is:

$$v_R = Ri = Ve^{-t/RC}$$

- The voltage across the capacitor is:

$$v_C = \frac{1}{C}\int i\,dt = V(1 - e^{-t/RC})$$



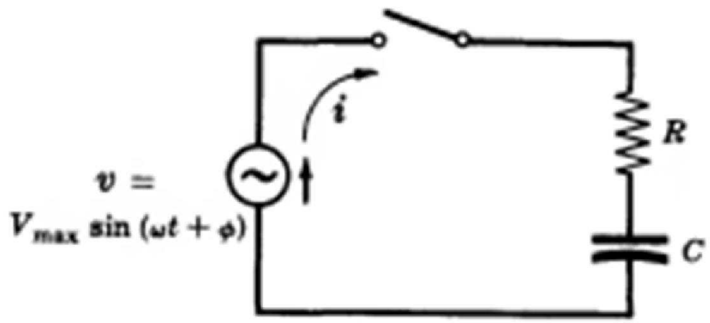
Transient-response is almost finished after 5τ, then steady state

1st Order R-C

AC

Alternating Current Transients

RC Sinusoidal Transient



$$Ri + \frac{1}{C}\int i\,dt = V_{\max}\sin(\omega t + \phi)$$

$$\left(D + \frac{1}{RC}\right)i = \frac{\omega V_{\max}}{R}\cos(\omega t + \phi)$$

$$i_c = ce^{-t/RC}$$

$$i_p = \frac{V_{\max}}{\sqrt{R^2 + (1/\omega C)^2}}\sin(\omega t + \phi + \tan^{-1} 1/\omega CR)$$

$$i = ce^{-t/RC} + \frac{V_{\max}}{\sqrt{R^2 + (1/\omega C)^2}}\sin(\omega t + \phi + \tan^{-1} 1/\omega CR)$$

Alternating Current Transients

RC Sinusoidal Transient

To determine the constant c, let $t=0$ then the initial current $i_0 = \frac{V_{\max}}{R}\sin\phi$. Substituting this into (*63*) and setting $t=0$, we obtain

$$\frac{V_{\max}}{R}\sin\phi \quad = \quad c(1) + \frac{V_{\max}}{\sqrt{R^2+(1/\omega C)^2}}\sin\left(\phi+\tan^{-1}1/\omega CR\right)$$

or

$$c \quad = \quad \frac{V_{\max}}{R}\sin\phi - \frac{V_{\max}}{\sqrt{R^2+(1/\omega C)^2}}\sin\left(\phi+\tan^{-1}1/\omega CR\right)$$

Substitution of c from (*65*) into (*63*) results in the complete current

$$i \quad = \quad e^{-t/RC}\left[\frac{V_{\max}}{R}\sin\phi - \frac{V_{\max}}{\sqrt{R^2+(1/\omega C)^2}}\sin\left(\phi+\tan^{-1}1/\omega CR\right)\right] + \frac{V_{\max}}{\sqrt{R^2+(1/\omega C)^2}}\sin\left(\omega t+\phi+\tan^{-1}1/\omega CR\right)$$

Examples

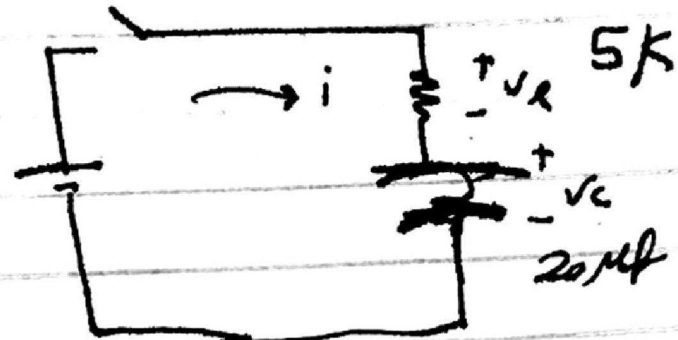
Example (4-sheet6)

A series RC circuit with R = 5000 ohms and C = 20 μf has a constant voltage V = 100 v applied at t = 0 and the capacitor has no initial charge. Find the equations of i, V_R and V_c.

Sol.

S closed

$$100 = i(5000) + \frac{1}{C}\int i\,dt \rightarrow \boxed{1} \quad 100\,V$$



$$100 = 5000\,i + \frac{1}{20\times10^{-6}}\int i\,dt$$

$$100 = 5000\,i + 50000\int i\,dt$$

نفاضل الطرفين للتخلص من التكامل

$$0 = 5000\frac{di}{dt} + 50000\,i$$

$$0 = (5000D + 50000)\,i$$

Example (4-sheet6)

$(D+10)\, i = 0$

it has only one solution (P.I)

$m + 10 = 0$

$m = -10$

$i = A e^{mt} = A e^{-10t}$

at $t = 0$ → sub in 1 ∴ $100 = 5000 i$ or $i = 100/500$

$i = 0.02A$

$$i = 0.02 e^{-10t}$$

$V_R = Ri = 5000 \times 0.02 e^{-10t} = 100 e^{-10t}$

$V_C = V - V_R$

$= 100 - 100 e^{-10t}$

Example (8-sheet 7)

A series RC circuit with R = 100 ohms and C = 25 µf has a sinusoidal voltage source v = 250 sin (500t + Ø) applied at a time when Ø = 0°. Find the current, assuming there is no initial charge on the capacitor.

Sol.

$$100\, i + \frac{1}{C}\int i\,dt = 250 \sin 500t$$

DR ⋲ $(D + 400)\, i = 1250 \cos(500t)$ — بعد التفاضل

Sol

$$i = c\,e^{-400t} + \frac{V_{max}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\omega t + \phi + \tan^{-1}\left(\frac{1}{\omega R C}\right)\right)$$

$$i = c\,e^{-400t} + 1.955 \sin(500t + 38.7^\circ)$$

at $t = 0$ $\longrightarrow$ $i = \frac{250}{100} \sin 0 = 2.5$

نعوض بـ $i = 2.5$, $t = 0$

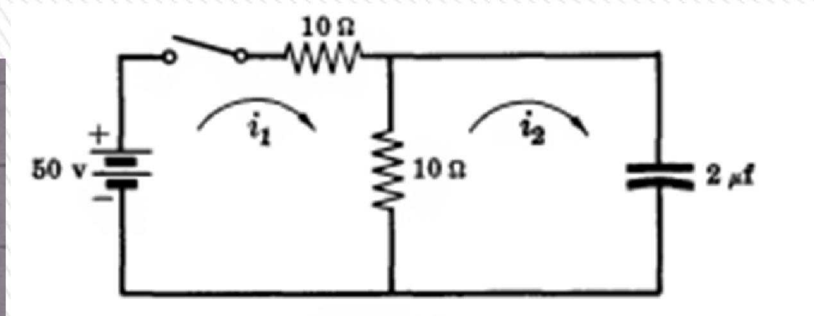
$$\therefore\ 2.5 = c\,e^{-400 \times 0} + 1.955 \sin(0 + 38.7)$$

$\therefore\ C = -1.22$

$$\therefore\ i = -1.22\,e^{-400t} + 1.955 \sin(500t + 38.7^\circ)$$

Example (6-sheet 7)

In the two-mesh network shown in Fig.4 the switch is closed at t = 0. Find the transient mesh currents i1 and i2 shown in the diagram, and the transient capacitor voltage Vc.



Loop 1

$$50 = 20 i_1 - 10 i_2$$

$$\therefore 0 = 20 D i_1 - 10 D i_2$$

or $$2 D i_1 = D i_2$$

بتفاضل الطرفين

Loop 2

$$0 = 10 i_2 - 10 i_1 + \frac{1}{C}\int i_2 \, dt$$

بتفاضل الطرفين

$$0 = 10 D i_2 - 10 D i_1 + \frac{1}{C} i_2$$

or $$-D i_1 + i_2 \left(D + \frac{1}{10C}\right) = 0$$

$C \rightarrow 2\times10^{-6}$

$$-D i_1 + i_2 (D + 5\times10^{4}) = 0 \rightarrow (2)$$

Example (6-sheet 7)

بالتعويض بالمعادلة ① في ②

$$\therefore -\frac{D i_2}{2} + (D + 5\times10^4) i_2 = 0$$

$$\text{or} \quad (D + 10^5) i_2 = 0$$

$$\therefore \text{sol} \quad i_2 = A e^{mt} = A e^{-10^5 t}$$

$$\text{at } t = 0 \longrightarrow \text{From eq } ② \quad \therefore 0 = 10 i_2 - 10 i_1$$

$$\therefore i_1 = i_2$$

والتعويض في i_1 أو i_2

$$\text{From } ① \quad 50 = 20 i_1 - 10 i_2 = 10 i_2 = 10 i_1$$

$$\therefore i_1 = i_2 = 5$$

$$\therefore \text{at } t = 0 \quad i_2 = 5 = A e^{0} \quad \therefore A = 5$$

$$\text{or} \quad \boxed{i_2 = 5 e^{-10^5 t}}$$

بالتعويض في المعادلة ①

$$50 = 20 i_1 - 10 \times 5 e^{-10^5 t}$$

$$\text{or} \quad \boxed{i_1 = \frac{5}{2} + \frac{5}{2} e^{-10^5 t}} \longrightarrow$$

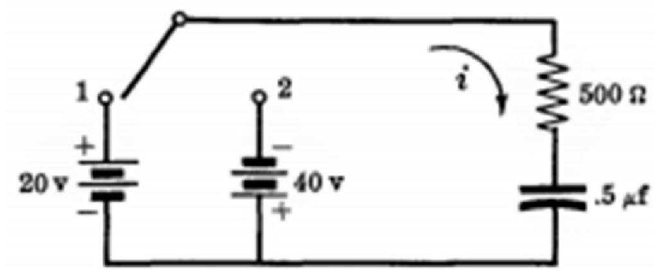
$$v_c = \frac{1}{C} \int i_2 \, dt$$

تكامل المعادلة i_2

$$v_c = 25 (1 - e^{-10^5 t})$$

Example (5-sheet 7)

In the RC circuit of Fig. 3 the switch is closed on position 1 at t=0 and after 1 TC is moved to position 2. Find the complete current transient.



at Position 1

$$20 = 500\,i + \frac{1}{C}\int i\,dt$$

بتفاضل :

$$0 = 500\frac{di}{dt} + \frac{1}{0.5\times10^{-6}}\,i$$

$$0 = 500\,Di + 2000000\,i$$

$$0 = Di + 4000\,i \qquad i(D + 4000) = 0$$

$$i_1 = A\,e^{-4000 t_1}$$

Example (5-sheet 7)

at $t=0$ → $i = 20/500 = 0.04 = A$

$\therefore\ i_1 = 0.04\, e^{-4000 t_1}$

يستمر هذا التيار لمدة

$250\ \mu sec = (1RC\quad) = 1TC$

after $1\,TC = 1RC = 250\ \mu sec$

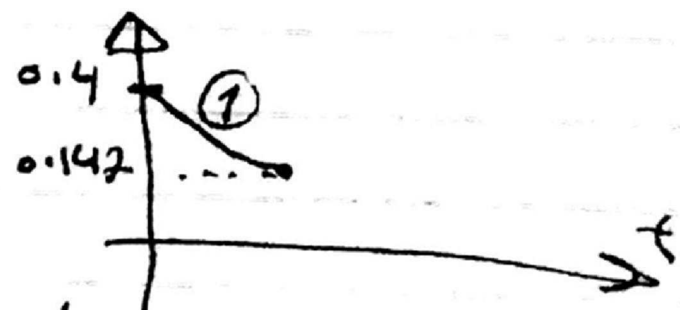
$i = 0.04 \times e^{-400 \times 250 \mu} = 0.147\ A$

Now switch moved to 2

$-40 + v_{c1} = 500\, i + \frac{1}{C}\int i\, dt$

$0 = 500\, di/dt + \frac{1}{0.5\times10^{-6}}\, i$

$i_2 = B\, e^{-4000(t-t_1)}$ *



Example (5-sheet 7)

$$\therefore\ v_C = 20\left(1-e^{-4000t}\right) = 20\left(1-e^{-4000\times (RC)}\right)$$

$RC = 250\times10^{-6}$

$1\tau =$ مدة زمنية

$$= 20\left(1-e^{-1}\right) = 12.65\ \text{Volt}$$

$$\therefore\ i_2 = B\,e^{-4000(t-t_1)}$$

لاحظ ان اتجاه قطبية C
مع اتجاه قطبية البطارية 40V

$$\text{at } t = t_1 \Rightarrow i = \frac{V_{total}}{R}$$

$$i = -\frac{\underset{\text{بطارية}}{40} + \underset{\text{مكثف}}{12.65}}{500} = -0.1053\,A$$

لاحظ السالب عشان ~~[illegible]~~ اتجاه تيار البطارية عكس
اتجاه i المرسوم

$$\therefore\ i_2 = -0.1053\,e^{-4000(t-t_1)}$$

Example (5-sheet 7)

at $t = t_1 = 250\,\mu sec$

لاحظ ازحنا مكان المفتاح لـ 2 لازال المكثف يحتفظ بالجهد عليه

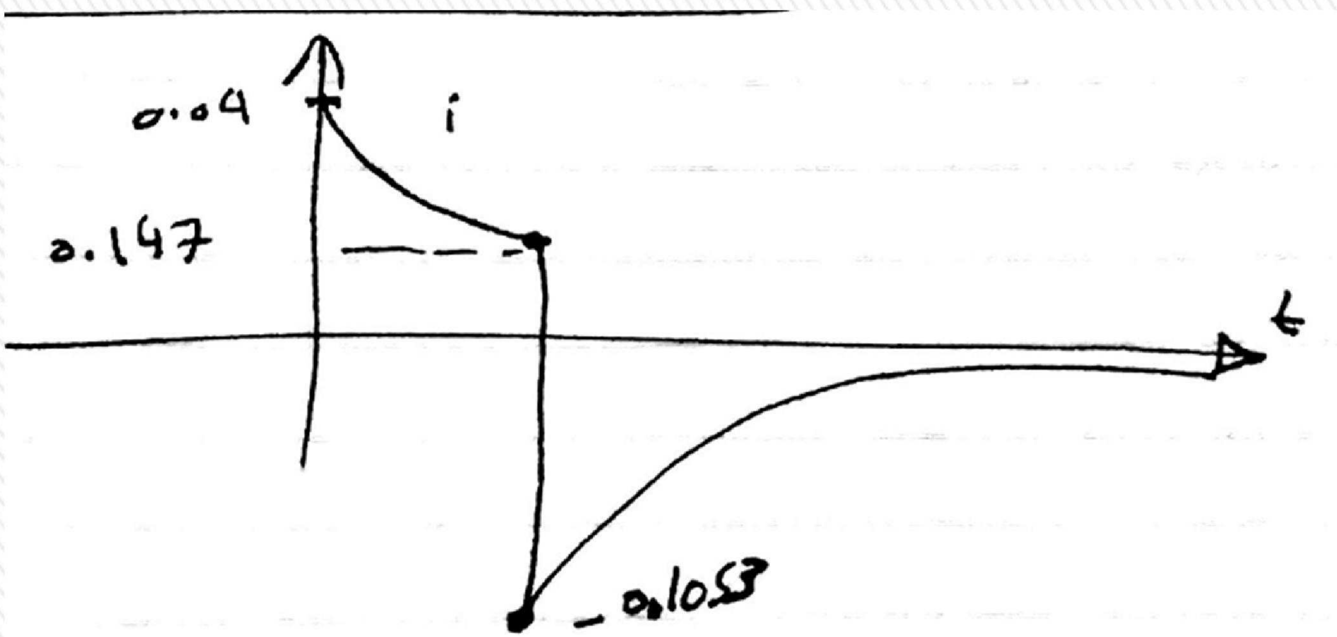
الى ال $\frac{1}{C}\int i\,dt = V_c$ وهو

$$\frac{(0.04)}{0.5\times10^{-6}}\,\frac{e^{-4000t}}{-4000} + K$$

$\therefore V_c = -20e^{-4000t} + K$

at $t = 0$ $K = +20$ $(V_c = 0,\ V_R = V)$

$\therefore V_c = 20(1 - e^{-4000t}) \rightarrow t = RC = 1\,TC$




Thank You